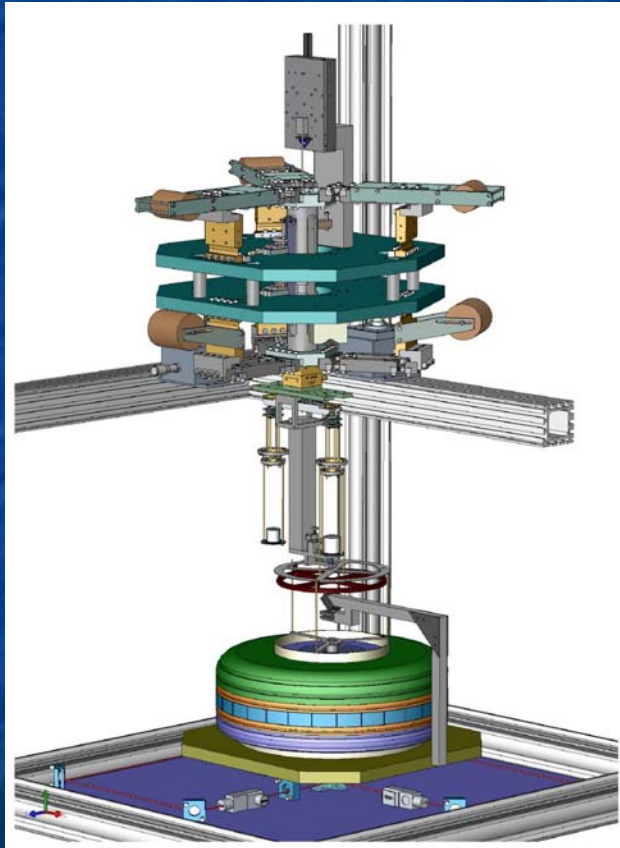


# Proposed changes to the SI , their impact on fundamental constants and other SI units .



Planck constant,  $h, e$   
LNE

**2002 CODATA RECOMMENDED VALUES OF THE FUNDAMENTAL CONSTANTS OF PHYSICS AND CHEMISTRY** NIST SP 959 (Apr/2005)  
 Values from: P. J. Mohr and B. N. Taylor, *Rev. Mod. Phys.* 77, 1 (2005). The number in parenthesis is the one-sigma (1  $\sigma$ ) uncertainty in the last two digits of the given value.

Quantity	Symbol	Numerical value	Unit
speed of light in vacuum	$c, c_0$	299 792 458 (exact)	$\text{m s}^{-1}$
magnetic constant	$\mu_0$	$4\pi \times 10^{-7}$ (exact)	$\text{N A}^{-2}$
electric constant $1/(\mu_0 c^2)$	$\epsilon_0$	$8.854 187 817 \times 10^{-12}$	$\text{F m}^{-1}$
Newtonian constant of gravitation	$G$	$6.674 2(10) \times 10^{-11}$	$\text{m}^3 \text{kg}^{-1} \text{s}^{-2}$
Planck constant	$h$	$6.626 0693 \times 10^{-34}$	J s
$h/(2\pi)$	$\hbar$	$1.054 571 6 \times 10^{-34}$	J s
elementary charge	$e$	$1.602 176 5 \times 10^{-19}$	C
fine-structure constant $e^2/(4\pi\epsilon_0\hbar c)$	$\alpha$	$7.297 352 568(24) \times 10^{-3}$	
inverse fine-structure constant	$\alpha^{-1}$	137.035 999 11(46)	
Rydberg constant $\alpha^2 m_e c/(2h)$	$R_\infty$	10 973 731.568 525(73)	$\text{m}^{-1}$
Bohr radius $a/(4\pi R_\infty)$	$a_0$	$0.529 177 2108(18) \times 10^{-10}$	m
Bohr magneton $eh/(2m_e)$	$\mu_B$	$9.27 400 949(80) \times 10^{-26}$	$\text{J T}^{-1}$

Quantity	Symbol	Numerical value	Unit
electron mass	$m_e$	$9.109 3826(16) \times 10^{-31}$	kg
proton mass	$m_p$	$1.672 621 71 \times 10^{-27}$	kg
proton-electron mass ratio	$m_p/m_e$	1836.152 672 89	
Avogadro constant	$N_A, L$	$6.022 1415 \times 10^{23}$	$\text{mol}^{-1}$
Faraday constant $N_A e$	$F$	96 485.3383	$\text{C mol}^{-1}$
molar gas constant	$R$	8.314 472	$\text{J mol}^{-1} \text{K}^{-1}$
Boltzmann constant $R/N_A$	$k$	$1.380 6505 \times 10^{-23}$	$\text{J K}^{-1}$
Stefan-Boltzmann const. $\pi^2 k^4/(60\hbar^3 c^2)$	$\sigma$	$5.670 400 \times 10^{-8}$	$\text{W m}^{-2} \text{K}^{-4}$
magnetic flux quantum $h/(2e)$	$\phi_0$	$2.067 833 73 \times 10^{-15}$	Wb
Josephson constant $2e/h$	$K_J$	$483 597.879 \times 10^9$	$\text{Hz V}^{-1}$
von Klitzing constant $h/e^2$	$R_K$	25 812.807 440	$\Omega$
electron volt $(e/C) \text{ J}$	eV	$1.602 176 5 \times 10^{-19}$	J
(unified) atomic mass unit $\frac{1}{12} m(^{12}\text{C})$	u	$1.660 538 86(28) \times 10^{-27}$	kg

A more extensive listing of constants is available in the references above and on the NIST Physics Laboratory Web site [physics.nist.gov/constants](http://physics.nist.gov/constants).

**NIST** National Institute of Standards and Technology  
 Technology Administration, U.S. Department of Commerce

## Fundamental Constants

Edwin Williams  
LNE, Guest Scientist  
& NIST

# CCM is asking:

- What system is best for the CCM and your metrology Community?  
The new SI in which we scale our system by fixing the values of  $e$ ,  $h$ ,  $N_A$  and  $k$  provides:
  - A system that is favorable to the mass community.
    - Agreement with other measurements of  $h$  and  $N_A$ .
  - A system more stable over time and more suitable for the expression of the values of the fundamental constants. (P. Mohr)
- What is needed to implement the new system?
  - Educate your community.
  - Implement the changes required to be consistent with new values of  $h$  and  $N_A$ .
- When? 2011 If 1ppm discrepancy resolved.
- Atomic mass and quantum electric standards are more stable, long term, than macroscopic mass standards

# What is the purpose of SI

- Provide a basis for a practical measurement system so that both science and industry can prosper
- We are being asked to simply choose the scales against which all measurements are made
  - We still have the same metric system but it won't drift and the scales will be clearer (have less uncertainty)



# Scientists can only disprove theories never prove them.

- The SI assumes that our present knowledge is valid but it is understood that the sciences upon which it is based must be tested.
- The SI simply provides a system where we can compare results from around the world.
- The adjustment of the fundamental constants is the most stringent test we make of the system.
- Defining  $e$ ,  $h$ ,  $N_A$  and  $k$  make it easier for everyone to see the points of disagreement.
- The SI must adjust as new theories become “present knowledge”.
  - JJ and QHE are driving the proposed redefinition.

# Example:

- Alpha, the fine-structure constant

$$\alpha^{-1} = \frac{2h}{\mu_0 c e^2} = \left[ \frac{2i}{\mu_0 c} (R_H) \right]$$

100 times less

# Avogadro constant from $h$ & $h/m(X)$

F. Biraben, et al.

Laboratoire Kastler Brossel

Laboratoire National de Metrologie et d'Essais

Institut National de Metrologie, CNAM

$$N_A = \left\{ \frac{K_J^2 R_K g^{(w)}}{4} \right\} \left\{ \frac{h}{m(^{87}\text{Rb})g^{(a)}} \right\} \left\{ \frac{g^{(a)}}{g^{(w)}} \right\} A_r(^{87}\text{Rb})M_u$$

Present

$$N_A = \left\{ \frac{1}{h} \right\} \left\{ \frac{c\alpha^2}{2R_\infty} \right\} A_r(m_e)M_u$$

# Quantum based systems

## 1998 codata

From 2001

Relative std. uncert.  $\times 10^{-9}$

Constant	Define Kg IPK	Define Kg $N_A$ or $m_u$	Define Kg h	Define V $2e/h$
IPK	exact	79 (IPK <sub>01</sub> )	78 (IPK <sub>01</sub> )	78 (IPK <sub>01</sub> )
$N_A$	79	exact	8	5
h	78	8	exact	4
e	39	4	2	4
$m_e$	79	2	8	5
$2e/h$	39 (K <sub>J-90</sub> )	3	2	exact
$m_p$	79	0.35	8	5
$m_u$	79 (amu)	exact	8 (amu)	5 (amu)

( ) Indicate an additional system or “representation”

## If we define a quantum kilogram today using CODATA 2002

Quantum based system

Relative std. uncert.  $\times 10^{-9}$

Constant	Define $m(K)$	Define $u, e, N_A, k$	Define $h, e, N_A, k$
$m(K)$	exact	170 (IPK <sub>2011</sub> ) (>20)	170 (IPK <sub>2011</sub> ) (>20)
$h$	170	1.4	exact
$N_A$	170	exact	exact
$e$	85	exact	exact
$m_e$	170	0.44	1.4
$2e/h$	85 (K <sub>J-90</sub> )	1.4 (K <sub>J-90</sub> )	exact
$m_p$	170	0.13	1.4
$u, m_u$	170 (amu)	exact (amu)	1.4 (amu)
$h/e^2$	.7(R <sub>K90</sub> )	1.4 (R <sub>K90</sub> )	exact
$F$	86	exact	exact
J in eV	85	exact	exact

( ) Indicate an additional system or “representation”

Redefining the kilogram is win-win for centuries into the future.



## *SI defined by fixing the values of a set of constants*

The International System of Units, the SI, is the system of units scaled so that the

(1) ground state hyperfine splitting transition frequency of the cesium 133 atom  $\Delta\nu(^{133}\text{Cs})_{\text{hfs}}$  is 9 192 631 770 hertz,

(2) speed of light in vacuum  $c_0$  is 299 792 458 meters per second,

(3) Planck constant  $h$  is  $6.626\ 069\ 3 \times 10^{-34}$  joule second,

$$m(\text{K}) = 1 \text{ kg } (1 \pm 2 \times 10^{-8})$$

(4) elementary charge  $e$  is  $1.602\ 176\ 53 \times 10^{-19}$  coulomb,

$$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^{-2} (1 \pm 7 \times 10^{-10})$$

(5) Boltzmann constant  $k$  is  $1.380\ 650\ 5 \times 10^{-23}$  joules per kelvin,

$$\text{Triple point H}_2\text{O} = 273.16 \text{ K } (1 \pm 2 \times 10^{-6})$$

(6) Avogadro constant  $N_A$  is  $6.022\ 141\ 5 \times 10^{23}$  per mole and

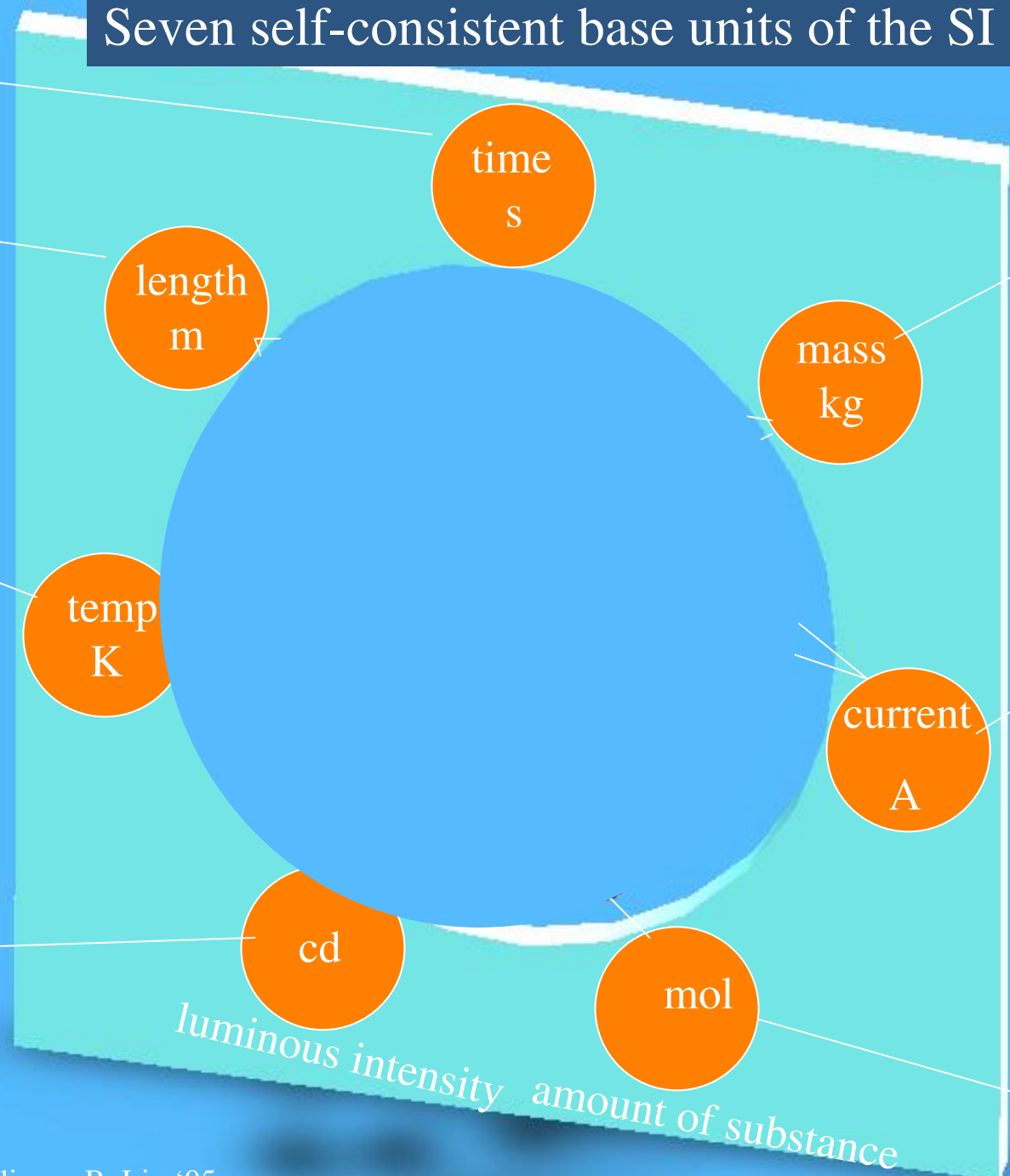
$$\text{Mole of C}_{12} = 12 \text{ g } (1 \pm 1.4 \times 10^{-9})$$

(7) spectral luminous efficacy of monochromatic radiation of frequency  $540 \times 10^{12}$  hertz  $K(\lambda_{555})$  is 683 lumens per watt.

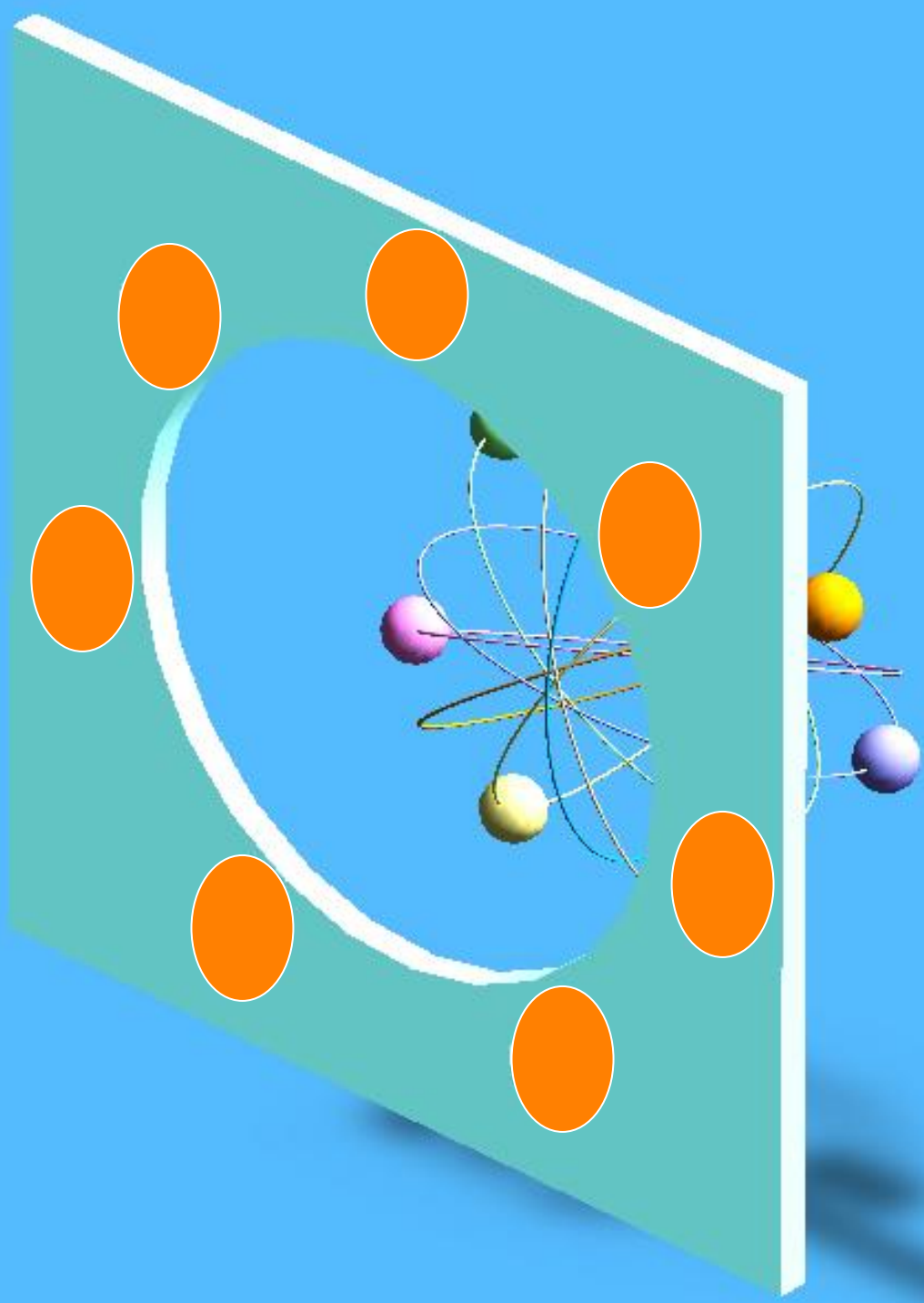
Seven self-consistent base units of the SI

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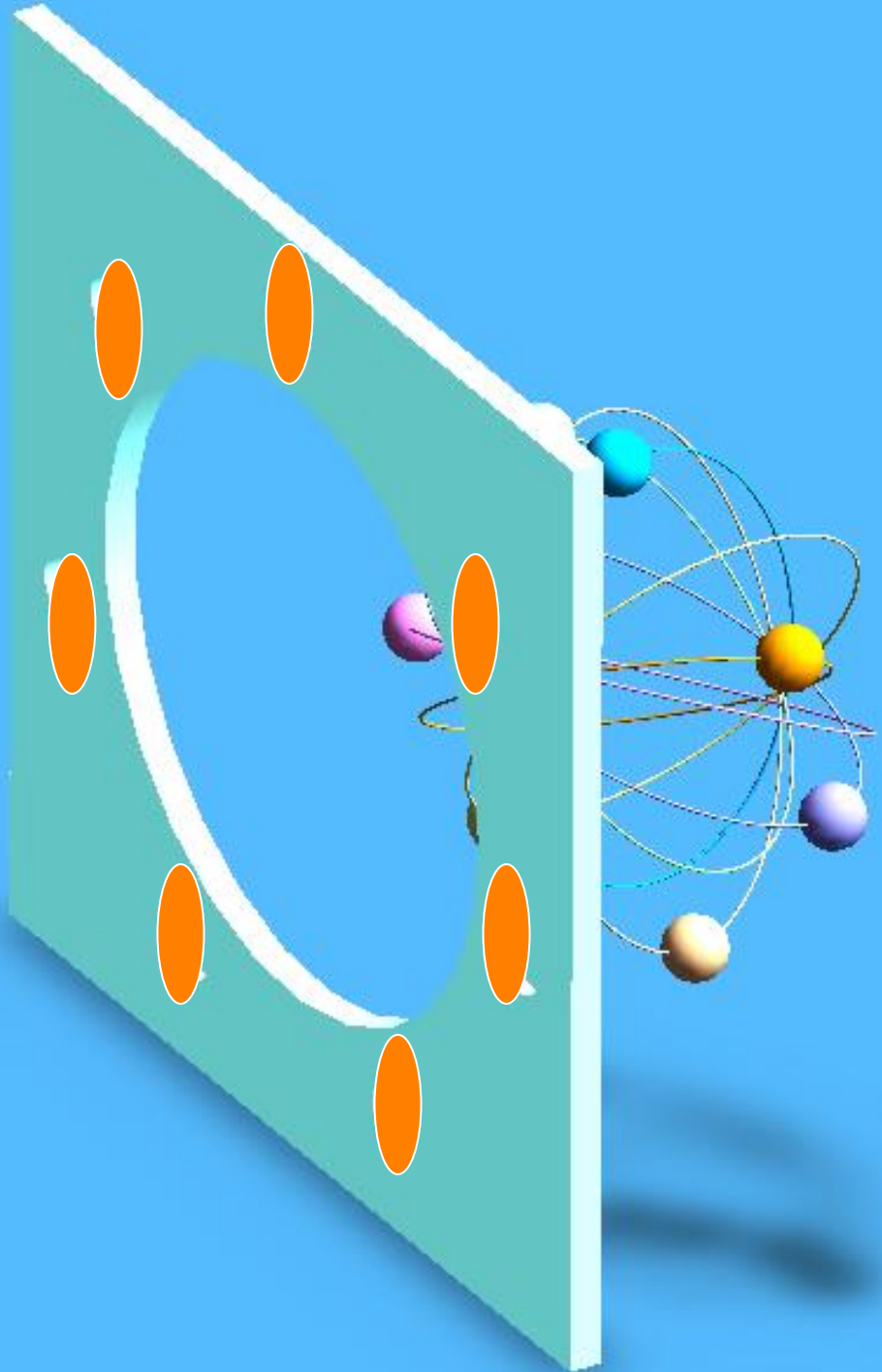


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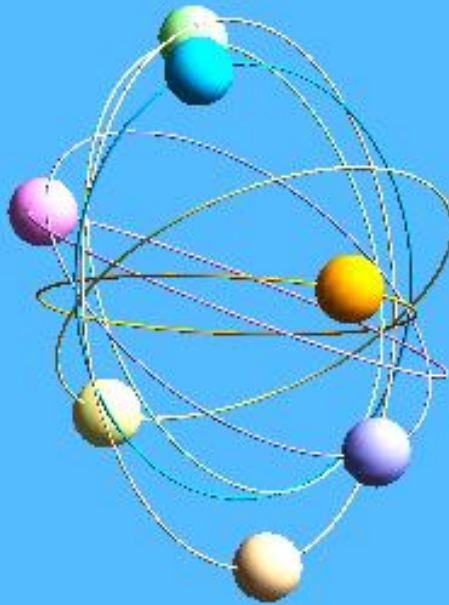
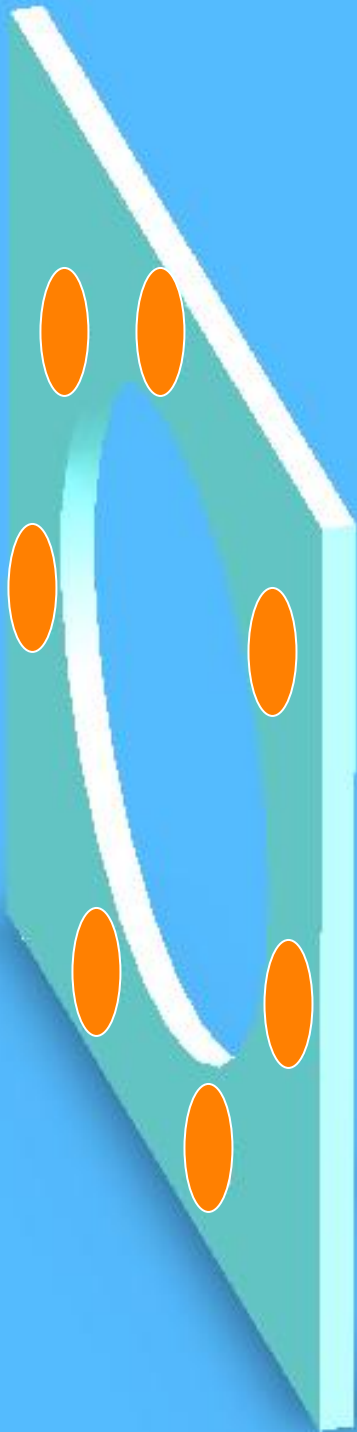
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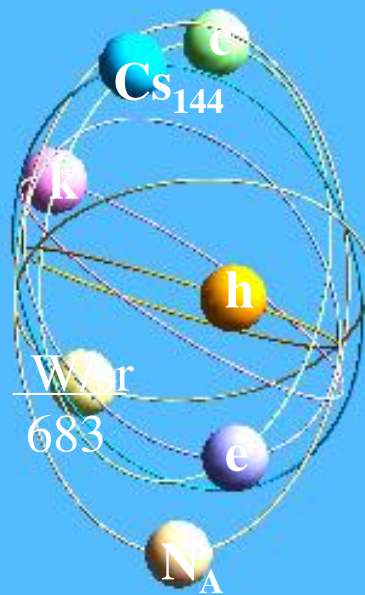
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- Much better fundamental constants
  - Mass of atoms connected to  $h$

Seven self-consistent constants based on physics



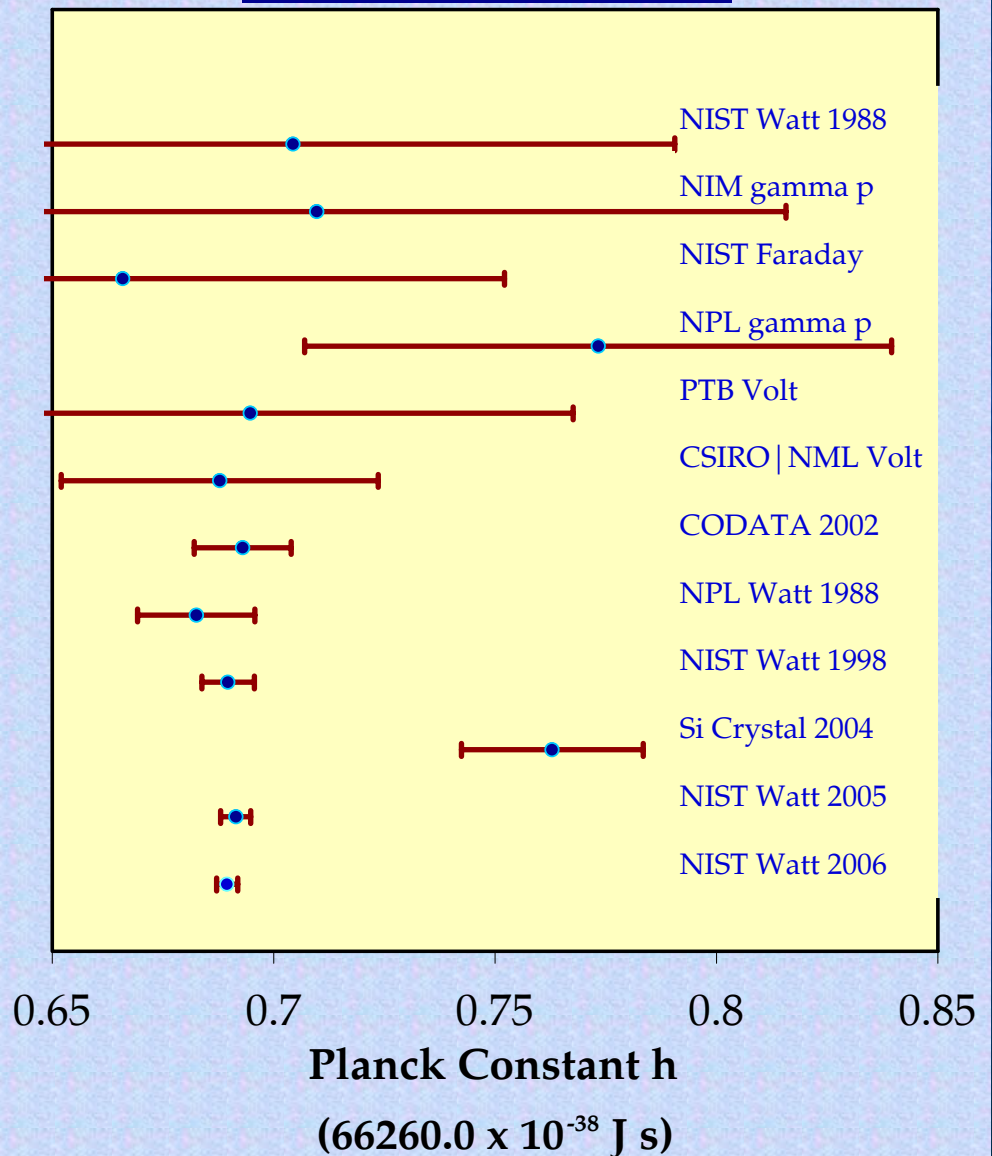
- Direct SI tie to constants
  - Atomic clocks
  - Josephson volt
  - QH Resistance
  - Temperature via  $k$

- Exact conversion factors
  - X-rays in eV

# Improving techniques

## The Planck constant

### Laboratory Values of the Planck Constant



# The new SI; An opportunity

- Provide the scientific community with an atomic based system.
  - Use to steer new macroscopic mass.
  - Science is the best source of new research.
- Opportunity to improve macroscopic mass dissemination.
  - Vacuum and inert gas environment.
  - Use a group of artifacts.
- Provide a opportunity for more fundamental research in mass metrology.
  - Need to compare Watt Balance and SI results.
  - Both vacuum masses and small masses tied to SI.



$k, N_A, e, h$

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Planck constant	$h$	$6.626 0693(11) \times 10^{-34}$	$\text{J s}$
$h/(2\pi)$	$\hbar$	$1.054 571 68(16) \times 10^{-34}$	$\text{J s}$
elementary charge	$e$	$1.602 176 53(14) \times 10^{-19}$	$\text{C}$
fine-structure constant $e^2/(4\pi\epsilon_0\hbar c)$	$\alpha$	$7.297 352 568(24) \times 10^{-3}$	
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Rydberg constant $\alpha^2 m_e c/(2h)$	$R_\infty$	10 973 731.568 525(73)	$\text{m}^{-1}$
Bohr radius $\alpha/(4\pi R_\infty)$	$a_0$	$0.529 177 2108(18) \times 10^{-10}$	$\text{m}$
Bohr magneton $e\hbar/(2m_e)$	$\mu_B$	$927.400 949(88) \times 10^{-26}$	$\text{J T}^{-1}$

No longer exact  
No longer exact

Exact  
Exact  
Exact

Improved

Quantity	Symbol	Numerical value	Unit
electron mass	$m_e$	$9.109 3826(16) \times 10^{-31}$	$\text{kg}$
proton mass	$m_p$	$1.672 621 71(23) \times 10^{-27}$	$\text{kg}$
proton-electron mass ratio	$m_p/m_e$	1836.152 672 61(85)	
Avogadro constant	$N_A, L$	$6.022 1415(10) \times 10^{23}$	$\text{mol}^{-1}$
Faraday constant $N_A e$	$F$	96 485.3383(82)	$\text{C mol}^{-1}$
molar gas constant	$R$	8.314 472(15)	$\text{J mol}^{-1} \text{K}^{-1}$
Boltzmann constant $R/N_A$	$k$	$1.380 6505(84) \times 10^{-23}$	$\text{J K}^{-1}$
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von Klitzing constant $h/e^2$	$R_K$	25 812.807 449(86)	$\Omega$
electron volt ( $e/C$ ) J	eV	$1.602 176 53(14) \times 10^{-19}$	$\text{J}$
(unified) atomic mass unit $\frac{1}{12}m(^{12}\text{C})$	$u$	$1.660 538 86(88) \times 10^{-27}$	$\text{kg}$

Improved  
Improved

Exact  
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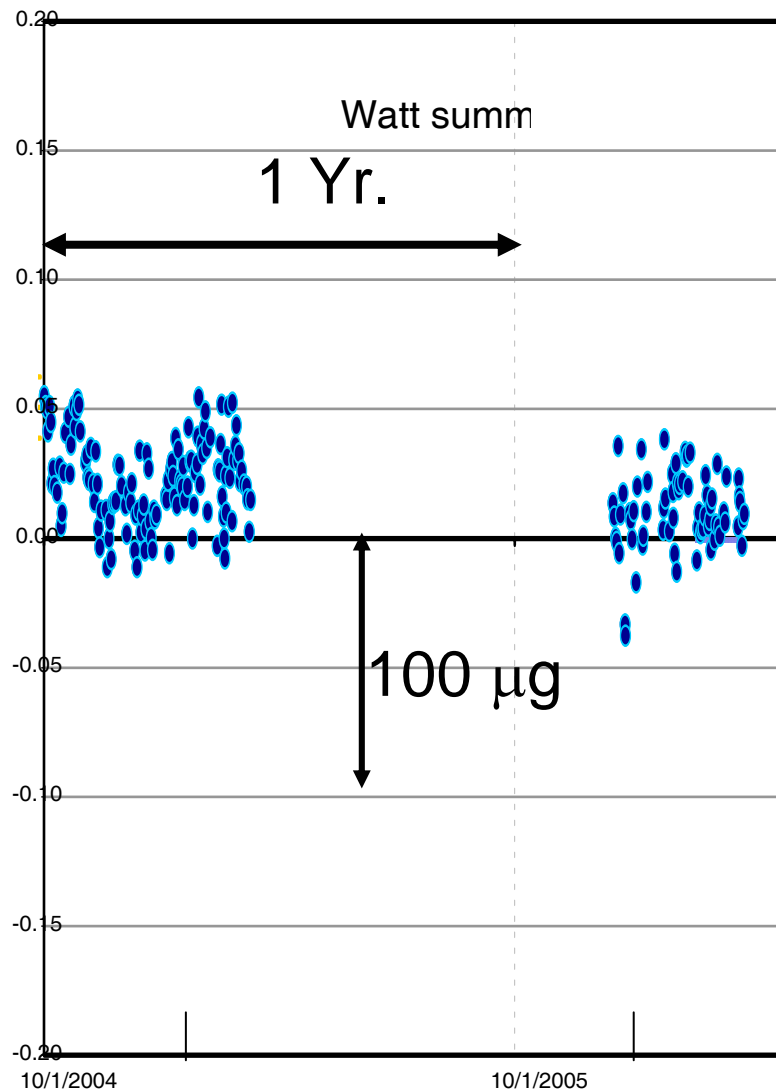
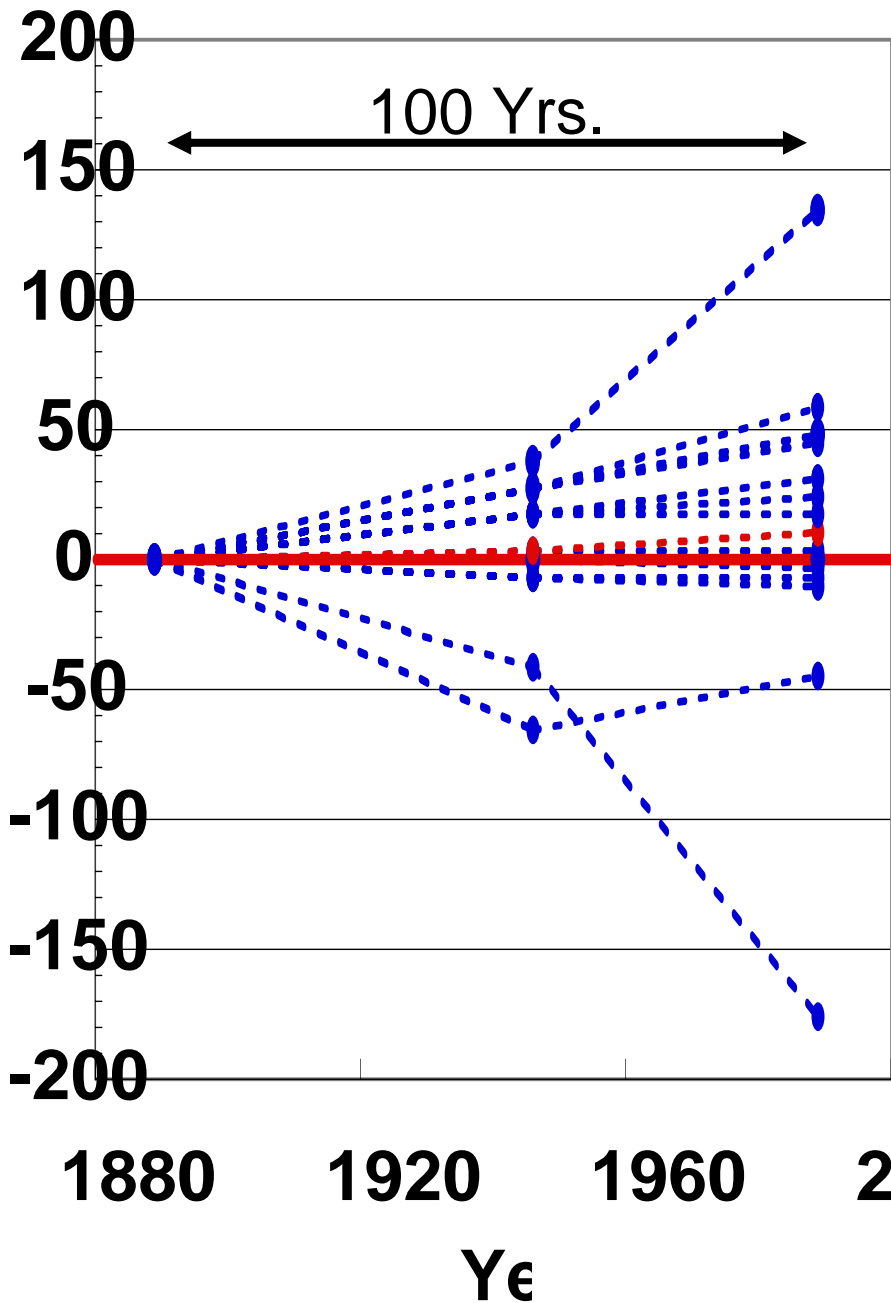
Exact  
Improved

A more extensive listing of constants is available in the reference given above and on the NIST Physics Laboratory Web site [physics.nist.gov/constants](http://physics.nist.gov/constants).

# Explaining the kilogram

The kilogram is the mass of  $6.022\,141\,5 \times 10^{26}$  idealized atoms, each of these atoms having a mass such that the Planck constant, the most important constant in quantum mechanics, has the specified value of  $6.626\,069\,3 \times 10^{-34}$  joule second.

Such atoms have a mass very close (within an uncertainty of 1.4 ng/g) to 1/12th the mass of  $^{12}\text{C}$ . This means that a mole of  $^{12}\text{C}$  weighs  $12 \times (1 \pm 1.4 \times 10^{-9})$  g.



# Why Now?

## Nobel prizes in physics

F. Bloch & E. Purcell	NMR	1952
A. Kastler	Spectroscopy	1966
Brian Josephson	Josephson effect	1973
Klaus von Klitzing	QHE	1985
Hans Dehmelt	Electron Traps	1989
Norman Ramsey	Separate osc. Fields	1989
Chu, Cohen-Tannoudji, Phillips	Atom cooling & trap	1997
Cornell, Ketterle, Wieman	BEC	2001
T. Hansch & J. Hall	Spectroscopy	2005





# Equations used to define or realize mass via from seven constants

$$E = h\nu = mc^2 \quad \text{Einstein}$$

$$mgv = \frac{U^2}{R} = S \frac{V_J^2}{R_H} \quad mgv = \frac{(h)^2}{ie^2} \left( \frac{nf}{2} \right)^2 ih \quad S \left( \frac{nf}{2} \right)^2 ih \quad \text{Watt balance}$$

Measure atomic mass unit  $u$  from XRCD

$$u = m(^{12}\text{C} / 12) = m_e / A_r(e) = 2hR_\infty / \alpha^2 c A_r(e)$$

$A_r(e)$  = atomic mass of electron

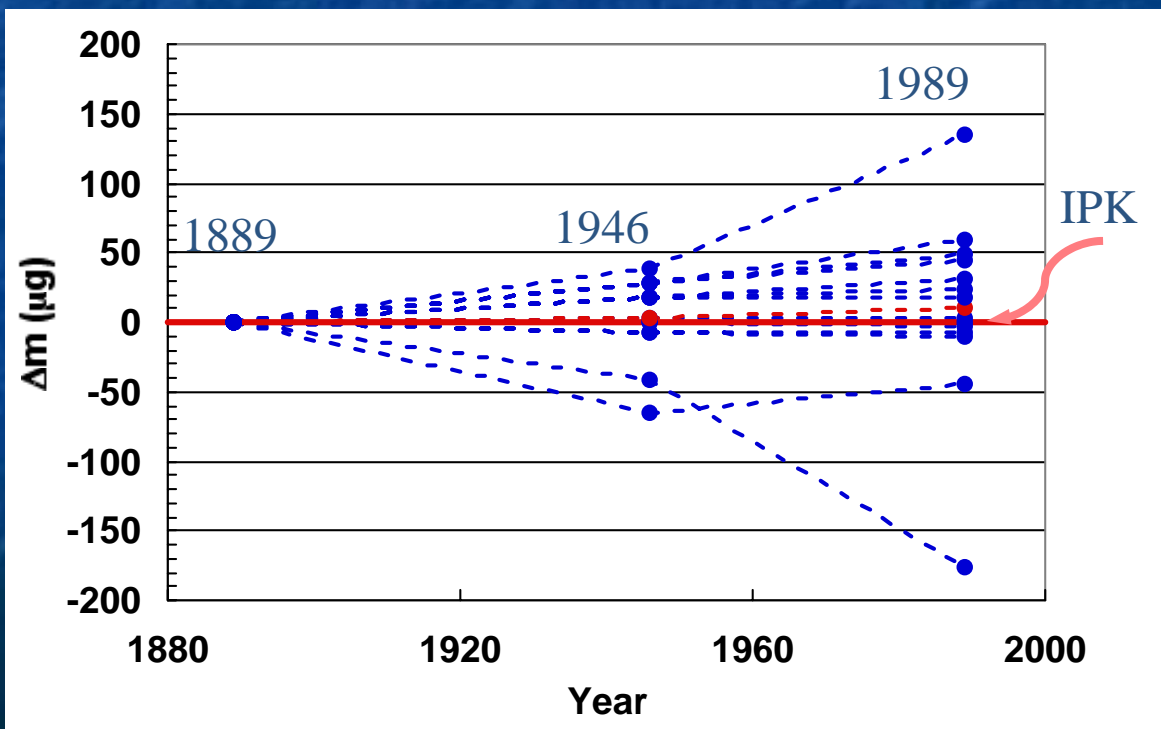
$R_\infty$  = Rydberg constant

*The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram*

CON:

- Can be damaged
- Gains mass from adsorption
- “Mass Correction” required after cleaning
- Long-term stability is inferred from other artifacts

PRO: Worked well!





## The ampere

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to  $2 \times 10^{-7}$  newton per meter of length.

Physics tells us  $F/l = (\mu_0 / 2\pi)(I^2/r)$

Magnetic permeability

$$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2 (1+\epsilon) \text{ where } \epsilon = 0 \text{ in 2011 } \sigma_r = 7 \times 10^{-10}$$

Maxwell Eqs.?



CODATA 1998--2002

Deadline: Dec 31, 2006

B. Taylor & P. Mohr

[www.nist.gov/physic/constants](http://www.nist.gov/physic/constants)



# Kilogram alternatives are related

$$u \leftrightarrow h$$

$$u = C_{12}/12 = X \{m\}_{SI} \{gv\}_{SI} / \{UI\}_{90}$$

Where

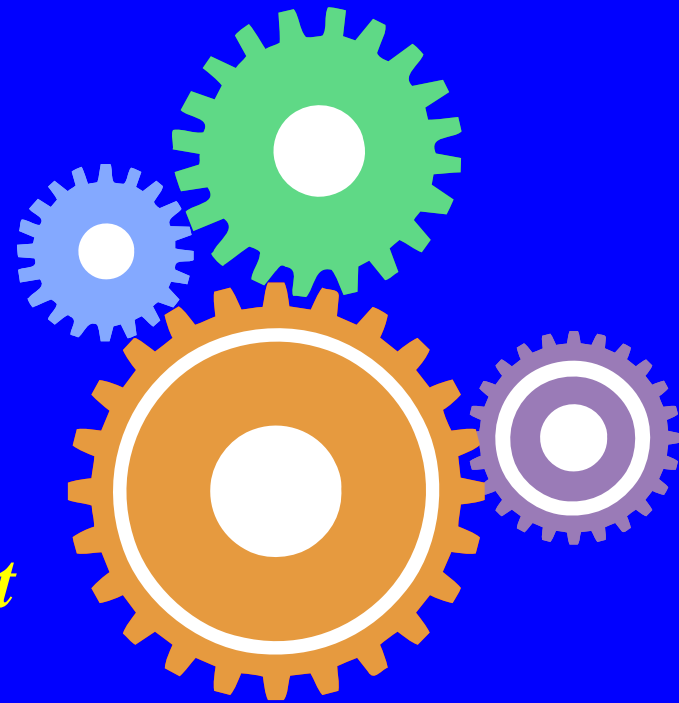
$$X = 8R_{\infty} / [c \alpha^2 (K_{J-90})^2 R_{k-90} A_r(e)].$$

and

$A_r(e)$  = atomic mass of electron

$R_{\infty} = \alpha^2 m_e c / (2h) = \text{Rydberg constant}$

*Uncertainty of  $X = 8 \text{ ppb}$*



# JE & QHE

Josephson Effect relation for the voltage

$U_J$  is:

$U_J = nf/K_J$  where  $n$  is a small integer,  $f$  an applied microwave frequency and

$$K_J = 2e/h$$

$$K_J = 483597.9? (.012) \text{ GHz/V } (20 \times 10^{-9})$$

CPEM 6/2006 Steiner, William

Quantum Hall effect

Von Klitzing constant

$$R_K = h/e^2 = \mu_0 c / 2\alpha$$

$$R_K = 25812.807? \Omega (0.7 \times 10^{-9})$$

