----- Forwarded message from ian704mills@btinternet.com -----
Date: Fri, 27 May 2011 22:15:21 +0100 (BST)
From: Ian Mills <ian704mills@btinternet.com>
Subject: Re: Redefinition of the kilogram
To: hill@math.gatech.edu, barry.taylor@nist.gov, i.m.mills@reading.ac.uk
Cc: "Mohr, Peter J." <mohr@nist.gov>, "terry.quinn@physics.org"
<terry.quinn@physics.org>, "Williams, Edwin R. Dr."
<edwin.williams@nist.gov>, "michael.kuehne@bipm.org"
<michael.kuehne@bipm.org>, "cthomas@bipm.org" <cthomas@bipm.org>,
"miller@lbl.gov" <miller@lbl.gov>, "himbert@cnam.fr"
<himbert@cnam.fr>, "alfornow@sbcglobal.net" <alfornow@sbcglobal.net>,
"paul.de.bievre@skynet.be" <paul.de.bievre@skynet.be>,
"athor@mech.kth.se" <athor@mech.kth.se>, "Marcus.Foster@csiro.au"
<Marcus.Foster@csiro.au>, "walter.emerson@wanadoo.fr"
<walter.emerson@wanadoo.fr>, "b.hibbert@unsw.edu.au"
<b.hibbert@unsw.edu.au>, "ingvar.johansson@philos.umu.se"
<ingvar.johansson@philos.umu.se>, "gcooper@pairmetrics.com.au"
<gcooper@pairmetrics.com.au>, "stephen.humphry@uwa.edu.au"
<stephen.humphry@uwa.edu.au>, "Ulrich.Feller@metas.ch"
<Ulrich.Feller@metas.ch>

Dear Ted, I attach my best attempt to answer your question. Perhaps Barry
Taylor may offer you a better answer. But a part of my answer is to
say to you
that some problems do not have a simple answer. That is the way that modern
science is.

Warm regards, Ian

===========================

Ian Mills FRS
President of the CCU
School of Chemistry
University of Reading
Reading RG6 6AD, U.K.

office email: i.m.mills@reading.ac.uk
office phone: +44 (0)118 378 8456
mobile: 07770 227678
home email: ian704mills@btinternet.com
home phone: +44 (0)118 987 2335

Question from Ted Hill:

What is your proposed introductory-level textbook definition of a kilogram? (including all the necessary pre-definitions, such as de Broglie, Planck, photon frequency etc)

imm's attempt to answer this:

If it is acceptable to define the metre as being such that the speed of light in vacuum *c* is exactly 299 792 458 m/s, it should be acceptable to define the kilogram as being such that the Planck constant *h* is exactly $6.626\ 068\ 96 \times 10^{-34}$ kg m^2 s^{-1}.

Now to expand on that. First, note that fixing the speed of light in vacuum to be exactly 299 792 458 m/s actually defines the unit metre per second, m/s, rather than the unit metre, m. The product of the number times the unit, $c = \{c\}[c]$, is the value of the speed of light, which is an invariant of nature and is not for us to choose. If we fix the unit $[c]$ in some other way, then we must do an experiment to determine the number $\{c\}$, and it will have an experimental uncertainty; this was the situation prior to 1983. If we fix the number $\{c\}$, then in effect we define the unit m/s so that the product shall be equal to the (invariant) value of the speed of light *c*; that has been the situation since 1983 when the new definition was adopted.

Since fixing the speed of light defines the m/s, then in order to define the metre it follows that we must also define the second. This is defined by the statement that the second is such that the period of the hyperfine transition in the caesium atom is exactly 1/(9 192 631 770) s. An alternative statement is that the second is such that the frequency of the hyperfine transition in the caesium atom is exactly 9 192 631 770 Hz, since Hz = s^{-1}.

Coming to the kilogram, similar comments apply. Fixing the Planck constant to be exactly $6.626\ 068\ 96 \times 10^{-34}$ kg m^2 s^{-1} actually defines the unit of action, kg m^2 s^{-1} = J s. Provided that we have already defined the metre and the second, then this defines the kilogram.

Further comment. I suspect that Ted will object that whereas the speed of light in vacuum *c* is a constant of nature that is acceptable to "college chemistry and physics students", the same is not true of the Planck constant *h*. If he makes that objection I do not agree with him. I believe the new young generation are more savvy than he gives them credit for. Of course they are not right up there with quantum mechanics, but it is not completely unfamiliar to them. (It was to me, when I was 18, and I am guessing it was to Ted. But this is the new generation, and they are a long way ahead of where we were at that age.) However if Ted persists in this objection, what you have to say is that the kilogram is defined to be equal to the mass of a prototype (or perhaps the average mass of a set of prototypes) kept at the BIPM, the mass of the prototype being actually itself defined in terms of a fundamental constant of nature called the Planck constant *h*, which plays a key role in our understanding of quantum mechanics in a manner similar to the way that the speed of light in vacuum plays a key role in our understanding of relativity, astronomy, and physics in general. The frontiers of modern physics, and of science in general, is a sophisticated subject, whether you like it or not. (You may even suggest to the student that it is a fascinating and exciting subject!)

I want to add yet another comment. It is important to distinguish between the definition of a unit, and the way in which we realise it in practice in the lab. To realise the unit means to do an experiment to make a measurement in terms of the unit in the laboratory. Advice on how to realise the definition is sometimes called a "*mise en pratique*" for the unit. The way in which a definition is realised may change as new experiments are developed even though the definition has not changed. This is why it is important to distinguish the *mise en pratique* from the definition.

To realise the definition of the second we must do an experiment that measures the period (or its reciprocal, the frequency) of the caesium transition and compares it with the time interval that we wish to measure. This may be done using a caesium atomic clock to compare the unknown time interval with the period of the clock, which we know by virtue of the definition of the second.

To realise the definition of the metre per second we must do an experiment that effectively measures the speed of light in vacuum c, and compares it with the speed that we wish to measure. Alternatively we may say that we must measure the wavelength of a laser of known frequency (that is where the need to know the second comes in) and compare that with the length that we wish to measure. This is all done by laser interferometry.

Finally to realise the definition of the kilogram we must do an experiment that effectively measures the value of the Planck constant h. This may be done with an apparatus called a watt balance (WB), which weighs a mass against an electromagnetic force. If we know the mass that we are using, the WB experiment measures the value of h, but if we have fixed the value of h by the definition then the WB reads out the value of the mass. The definition may also be realised by a so-called x-ray crystal density (XRCD) experiment which effectively involves measuring the mass of a single silicon atom and comparing it with the unknown mass that we wish to measure. Theory shows that the value of the mass of a silicon atom is closely related to the value of the Planck constant, so that we may calculate the one from the other, or *vice versa*. Thus either the WB or the XRCD experiment may be used to realise the definition of the kilogram; they should agree.

Our "college chemistry and physics student" should be aware that modern quantum physics is indeed a sophisticated subject, and to define a set of units for the world appropriate to the needs of the 21st century is not a simple matter. Quantum physics is, however, a subject that for me has been the fascination of my life. I love it. I do not actually believe there are simpler answers to Ted's question, but our student should not be put off by this. He should be challenged by it! I believe the students of today will accept that challenge.

All these ideas are carefully presented in our paper by Mills, Mohr, Quinn, Taylor and Williams currently in press for Philosophical Transactions of the Royal Society.

Ian Mills, 27 May 2011