Subject: Re: Redefinition of kilogram
Date: 5/4/07 8:54:54 PM
From: "Barry Taylor"
To: "Ted Hill" , i.m.mills@reading.ac.uk, mohr@nist.gov,
terry.quinn@physics.org, edwin.williams@nist.gov
Cc: ron.fox@physics.gatech.edu

Dear Ted,

I will try to answer your questions, but perhaps you are making the whole business more complex than necessary. Further, a telephone conversation may be more useful than written exchanges. See the interspersed red text, which I hope you find helpful.

Regards,

Barry

At 04:50 PM 5/4/2007, Ted Hill wrote:
Gentlemen,

I just returned from an extended truip abroad and am re-reading your 2006 Metrologia article (Vol 43, p
227-246). I still find it confusing in places, and perhaps some of you do also - one author told me that kappa
is changing in time while another told me it is NOT changing in time, and at one point in my conversation with
Dr. Mohr, he told me that the same symbol N_A might mean different things in different parts of that paper.

Your paper stresses the importance of communication between scientists and metrologists, and it is in that spirit
that I am asking for further explanations. If the proposal detailed in your paper contains significant shortcomings
or errors, that should be brought to the attention of both communities, especially the readers of Metrologia.

Perhaps I am wrong, but if you could answer the following basic questions, that might be a good place to start.

1. Exactly what does the symbol N_A mean in that paper?

Imagine the following. The SI is as currently defined, it is 1 January 2011, and the 2010 CODATA constants adjustment has just been completed. At its meeting in October 2011 the CGPM decides to adopt new definitions of the kilogram, ampere, kelvin and mole that links these four units to exact values of h, e, k, and N_A, respectively. The actual wordings of the definitions are unimportant -- they can be explicit unit definitions like the present definition of the meter, which links the meter to a fixed value of c, or an explicit constant definition, which for the meter might read "The meter, the SI base unit of length, is such that the speed of light in vacuum c is exactly 299 792 458 meters per second."

Now, what values of h, e, k, and N_A should the CGPM choose? Obviously, to maintain continuity with the current kilogram, ampere, kelvin, and mole, they should choose the 2010 CODATA recommended values of h, e, k, and N_A since these values are the most accurate available in SI units based on our knowledge as it existed on 31 December 2010. However, the CGPM will take these values to be exact, i.e., with no uncertainty, and hence these values will never change forever more, just like the current value of c is exact and will never change. Of course, the values chosen will have to have enough digits to keep any discontinuity introduced by using truncated values to a negligible level, which for the sake of argument we will assume to be 11 digits (10 may be adequate or 12 may be needed; it really doesn't matter). The symbols for these constants will remain the same, because the value of a quantity is independent of the unit in terms of which its value is expressed. However, the numerical value of a quantity depends on the unit, and the newly defined kilogram, ampere, kelvin, and mole are actually different units from the currently defined kilogram, ampere, kelvin, and mole. Hence in principle, they should have some distinguishing mark (i.e., a different symbol). Nevertheless, this was not done when the meter was redefined in terms of c, and hence the current symbols for the kilogram, ampere, kelvin, and mole, that is, kg, A, K, and mol, will be retained for the newly defined units just as the symbol m was retained for the newly defined meter in 1983.

(Is it perhaps the 2002 NIST range of values for Avogadadro's constant (i.e., a variable with that specified mean and standard deviation), or is it the central value of that range (i.e., a constant fixed in 2002), or is it the current best experimental estimate of one of those? Or something else?)

2. By your definitions, would the exact mass (in grams) of one atom of carbon-12 be changing in time?

The mass of an atom of carbon-12 is an invariant of nature and hence its value does not change. What does change is our knowledge of that value when expressed as a number times a unit. In the new SI where c, h, e, k, and N_A have exactly known values, one can show that the mass of the carbon-12 atom is given by

m(12^C) = (24R_inf h)/[c alpha^2 A_r(e)], (1)

where R_inf is the Rydberg constant, alpha^2 is the square of the fine-structure constant, and A_r(e) is the relative atomic mass of the electron. Now h and c are exactly known but R_inf, alpha, and A_r(e) are not, hence their numerical values will change as new experimental and theoretical results become available. Thus, the numerical value of m(12^C) expressed in the new kilogram will change. Based on the 2006 CODATA recommended values, if the SI were redefined today as we propose, the relative uncertainty of m(12^C) would be approximately 1.4 parts in 10^9 due to the uncertainties of R_inf, alpha^2, and A_r(e).

3. Isn't kappa essentially a new fundamental constant?

(it seems to be the crucial link between your new proposed numerical definitions of fundamental constants and the physical world of real atoms - e.g. via amu/carbon-12)

I suppose one could call (1 +kappa) a new fundamental constant if one wishes to do so, but one should keep in mind that it is really just a combination of well known constants and writing the combination as (1 + kappa) is for convenience. Note that in the new SI we have

(1 + kappa) = (2R_inf N_A h/)[c alpha^2 A_r(e) M_u], (2)

where N_A is the Avogadro constant and M_u is the molar mass constant, equal to 10^-3 kg/mol exactly. Thus, the relative uncertainty of (1 + kappa) is the same as that of m(12^C) since both N_A and M_u have no uncertainty in the new SI. Note that its numerical value of (1 + kappa) will be zero at the time of adoption of the New SI, but this will change with time.

4. What is your proposed introductory-level textbook definition of a kilogram (cf your Table 1)?

(including all the necessary pre-definitions, such as de Broglie, Planck, photon frequency etc)

I am not in the business of writing introductory textbooks -- I will leave that to others. All I will say is that we do not believe any of the proposed new definitions are any more complex than the current definitions of some of the SI base units.

5. What are your proposed introductory-level textbook definitions for one amu and for kappa?
The definition of the unified atomic mass unit is unchanged -- it remains m(12^C)/12. Its value in (new) kg at any point in time can be calculated from equation (1) above.

6. Your proposal for redefinition of the mole (e.g., in mol-1, mol-2 etc), the number 6.0221415 X 10^23, requires
79 bits, and therefore can not even be entered on a digital computer that uses IEEE Standard binary
or hexadecimal double-precision arithmetic. Hence any accuracy attained in adopting that particular fixed value would apparently be immediately lost in performing any scientific calculation based on the value for a mole (or Avogadro's number, etc).
[The same problem would arise with your proposed values for Planck, Boltzman and elementary charge constants.]
Use Fortran and exponential notation.

Thank you for your feedback in the past - I look forward to your responses to these questions.

Regards

Ted Hill

****
Barry N. Taylor
NIST, Bldg. 221, Rm. A255
100 Bureau Drive, Stop 8420
Gaithersburg, MD 20899-8420
USA
Tel.: 301-975-4220
Fax: 301-990-1350
Web: <http://physics.nist.gov/cuu>
<http://physics.nist.gov/cuu>
http://physics.nist.gov/cuu <http://physics.nist.gov/cuu>
****